Time derivatives

1 Computing time derivatives

In some cases it might be necessary to generate a time derivative of some signal on an analog computer. It should be noted that in general integration should always be favored instead of differentiation as the latter tends to increase noise substantially. However, when one wants to implement e. g. a PID-controller, the proportional (P) and integral (I) part are easy but the derivative (D) turns out to be a bit tricky. In this application note two approaches to compute approximate solutions for $\frac{\mathrm{d}}{\mathrm{d}t}f(t)$ are shown.[1]

1.1 A simple signal generator

First, a simple signal generator is described the outputs of which serve as simple, yet suitable test inputs to the differentiator circuits. This circuit yields a cosine-, a square-, and a triangle-signal as shown in figure 1. The schematic is straightforward: The upper subcircuit generates a $\cos(\omega t)$ signal using two Z-diodes to prevent an overload.[2] This signal then controls a comparator switching switches between $+1$ and -1 yielding a square wave signal. This signal is then fed into a third integrator yielding a triangle signal.[3] The coefficient a should be set to get a nice and large triangle signal without overload.[4]

1.2 Direct time derivative

The first circuit shown is pretty straightforward: The basic idea is to swap the input resistor and feedback capacitor of an integrator as shown in figure 2. The capacitor and the coefficient

[1]More details on the techniques described below can be found in [Giloi et al. 1963, pp. 160 ff.], [Sydow 1964, pp. 46 f.], and [Ulmann 2023/1, pp. 89 ff.].

[2]In fact, both integrators, especially the second one, will still experience an overload but not so much as to distort the signal too much.

[3]Although this circuit is far from perfect, the integrator yielding the triangle output signal is stable enough to allow for run times of several minutes before drifting away too far in the case of a THAT.

[4]In systems which do not have comparators, an external signal generator should be used.

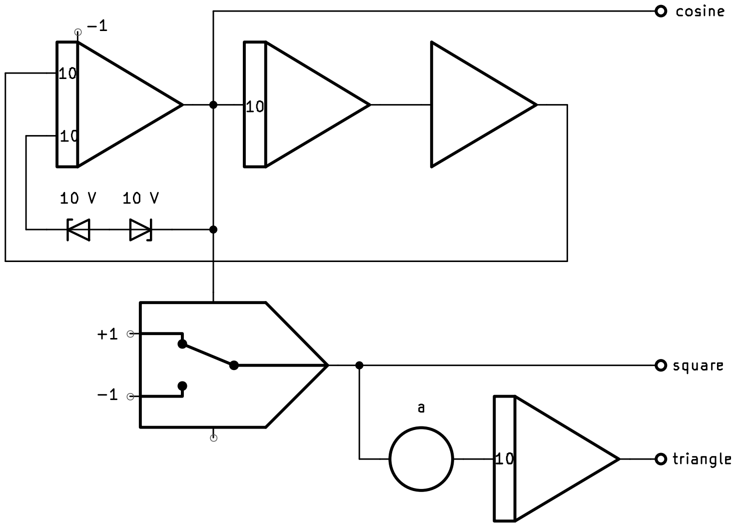

Figure 1: Simple signal generator

b are chosen manually to yield the best possible time derivative for a given signal type. Picture 3 shows the behavior of this setup.[5]

1.3 A better circuit

A much better implementation of a time derivative which does not require additional passive elements and works really great on modern systems is shown in figure 4. The parameter α is of central importance, controlling the frequency response of the circuit and should be as large as possible, meaning that there should be no overshoots or instabilities in the output signal of the circuit. In typical setups $\alpha \approx 0.85$. The behavior of this circuit is shown in figure 5.[6] The weight of 10 at the integrator input is due to the frequency of the signal

[5]The quality of the output signal could be improved by adding a small series resistor between the capacitor and the summing junction of the inverter/summer. The screen shots were taken from a Gould OS4200 digital storage oscilloscope which explains the rather uneven exposure of the pictures.

[6]This circuit can be used directly as the D-part of a PID controller.

generated by the simple circuit shown in figure 1. At even higher frequencies even larger weights might be necessary.

A rather handwavy explanation of the circuit goes like this: The summer, followed by an inverter and a (rather large) positive feedback by the coefficient α approximates a non-inverting open amplifier with the sign flipping integrator in its main feedback loop. It will thus yield an output signal $y(t)$ satisfying $\int y(t)\,\mathrm{d}t = f(t)$ with $f(t)$ denoting the input signal. The only problem here is the instability of the algebraic loop consisting of the summer and inverter when α becomes too large. From a mathematical point of view $\alpha \to 1$ would be desirable. Practical experiments show that the circuit tends to get instable for $\alpha > 0.85$, though.

A more thorough explanation takes a look at the output signal $y(t)$ at the inverter:

$$y(t) = f(t) - \int y(t)\,\mathrm{d}t + \alpha y(t)$$

Taking the time derivative on both sides yields

$$\dot{y}(t) = \dot{f}(t) - y(t) + \alpha\dot{y}(t).$$

Bringing $\dot{y}(t)$ to the right and $y(t)$ to the left then yields

$$\begin{aligned} y(t) &= \dot{f}(t) - \dot{y}(t) + \alpha\dot{y}(t) \\ &= \dot{f}(t) + (\alpha - 1)\dot{y}(t). \end{aligned}$$

Now let $\alpha \to 1$, so that $y(t) \approx -\dot{f}(t)$.

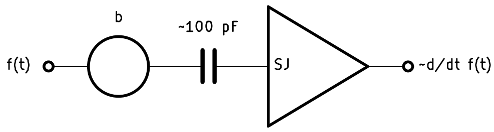

Figure 2: Direct differentiation circuit

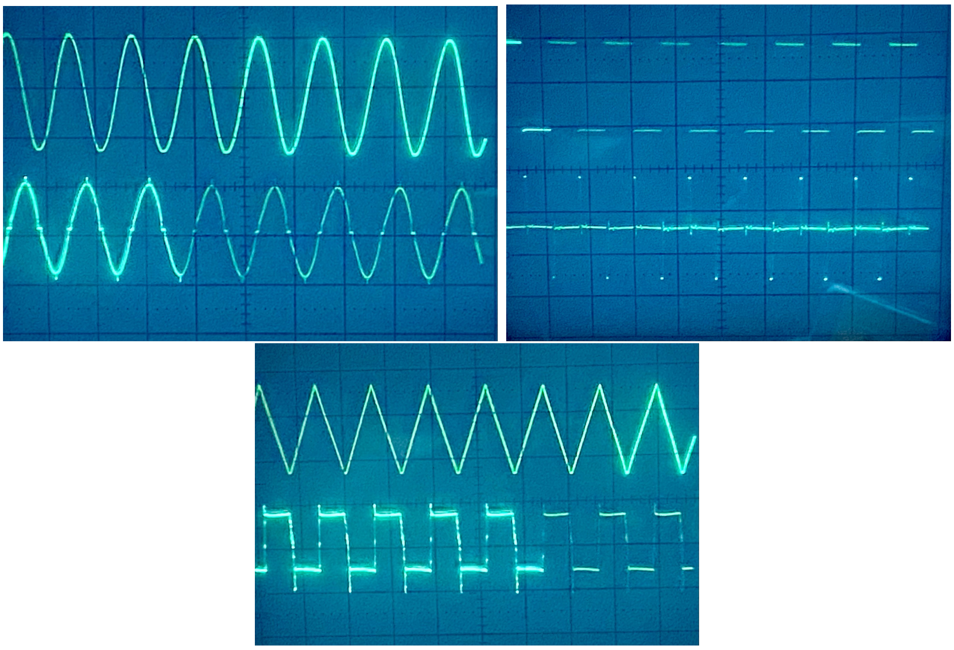

Figure 3: Behavior of the direct differentiation circuit applied to a cosine, a square wave, and a triangle signal

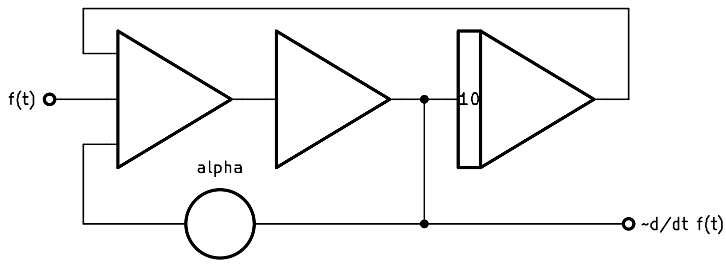

Figure 4: Clever time derivative circuit

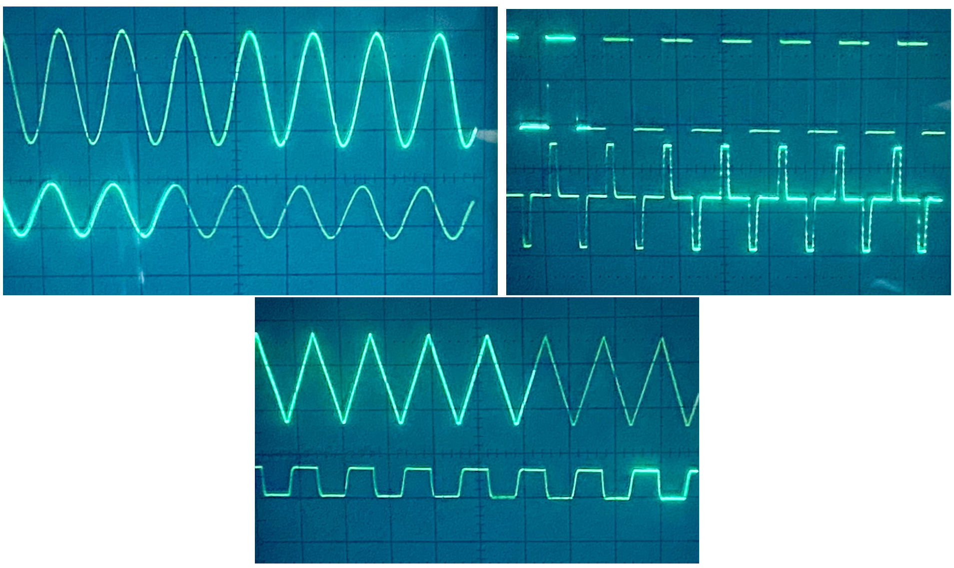

Figure 5: Behavior of the differentiation circuit shown in figure 4 applied to a cosine, a square wave, and a triangle signal

References

[Giloi et al. 1963] Wolfgang Giloi, Rudolf Lauber, *Analogrechnen*, Springer-Verlag, 1963

[Sydow 1964] Achim Sydow, *Programmierungstechnik für elektronische Analogrechner*, VEB Verlag Technik Berlin, 1964

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